Supplemental Explanation of Theorem 4 of Theoretical Computer Science, Vol.838, pp.238–249, entitled "Exact algorithms for the repetition-bounded longest common subsequence problem"

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October 12, 2023

- The following part in the statement of the theorem

  > Let $Z_{1..h} = \langle z_1, z_2, \ldots, z_h \rangle$ be any repetition-bounded longest common subsequence of $X_{1..i}$ and $Y_{1..j}$ satisfying an occurrence vector $\mathbf{v}$. Then the followings are satisfied:

  must have been replaced with the following.

  > There is a repetition-bounded longest common subsequence $Z_{1..h} = \langle z_1, z_2, \ldots z_h \rangle$ of $X_{1..i}$ and $Y_{1..j}$ satisfying an occurrence vector $\mathbf{v}$ and the following properties:

- In (1) and (2), "$z_h = s_p$" indicates that z_h comes from the common symbol s_p of x_i and y_j. However, this expression might be inappropriate, since even if x_i and y_j are both s_p, the expression $z_h = s_p$ could include a situation that $z_h = s_p$ comes from another pair of positions, say, $x_{i'}$ and $y_{j'}$ for $i' > i$ and $j' > j$. Thus, please read that $z_h = s_p$ here means that $z_h = x_i = y_j$.

- In (2), the situation $z_h = x_i = y_j$ is only considered. Here we note that any repetition-bounded longest common subsequence satisfying $z_h \neq x_i$ has length at most the length of a repetition-bounded longest common subsequence satisfying $z_h = x_i$:

  The condition $z_h \neq x_i$ implies that $Z_{1..h}$ is a repetition-bounded longest common subsequence of $X_{1..i-1}$ and $Y_{1..j-1}$ satisfying $\mathbf{v}$ (which can be shown by a similar discussion to the proof of (2) of the theorem). Assume that $Z_{1..h}$ satisfies $z_h \neq x_i$ and $z_k = s_p = x_{i'} = y_{j'}$ for $k < h$, $i' < i$, and $j' < j$. Then, we can remove z_k from $Z_{1..h}$ and instead add $s_p = x_i = y_j$ at the end without decreasing its length or violating the vector $\mathbf{v}$. In the obtained common subsequence $Z'_{1..h-1}$, the part $Z'_{1..h-1}$ satisfies a vector $\mathbf{v}|_{p=v_p-1}$, and also $z_h = x_i$ holds. The case that such z_k is not included in $Z_{1..h}$ is treated in (3) of the theorem. Hence, it is enough to consider repetition-bounded longest common subsequences satisfying $z_h = x_i$ in (2) of the theorem.